

Lepton universality test in the photoproduction of e^-e^+ versus $\mu^-\mu^+$ pairs on a proton target

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In view of the significantly different proton charge radius extracted from muonic hydrogen Lamb shift measurements as compared to electronic hydrogen spectroscopy or electron scattering experiments, we study in this work the photoproduction of a lepton pair on a proton target in the limit of very small momentum transfer as a way to provide a test of the lepton universality when extracting the proton charge form factor. By detecting the recoiling proton in the $\gamma p \rightarrow l^- l^+ p$ reaction, we show that a measurement of a ratio of $e^-e^+ + \mu^-\mu^+$ over e^-e^+ cross sections with an absolute precision of 7×10^{-4} , would allow for a test to distinguish, at the 3σ level, between the two different proton charge radii currently extracted from muonic and electronic observables.

Recent extractions of the proton charge radius R_E from muonic hydrogen Lamb shift measurements [1, 2] are in strong contradiction, by around 7 standard deviations, with the values obtained from energy level shifts in electronic hydrogen [3] or from electron-proton elastic scattering [4, 5] :

$$\mu p [2] : \quad R_E = 0.8409(4) \text{ fm} , \quad (1)$$

$$ep [3] : \quad R_E = 0.8775(51) \text{ fm} . \quad (2)$$

This so-called "proton radius puzzle" has triggered a large activity and is the subject of intense debate, see e.g. [6–8] for recent reviews, and references therein. Lepton universality requires the same radius to enter the electronic and muonic observables. If the different R_E extractions cannot be explained by overlooked corrections, it would point to a violation of electron-muon universality. Several scenarios of new, beyond the Standard Model, physics have been proposed by invoking new particles which couple to muons and protons, but much weaker to electrons, see e.g. [9–13]. Such models would also lead to large loop corrections to the muon's anomalous magnetic moment, $(g-2)_\mu$, which presently displays a 3σ deviation between experiment and its Standard Model prediction, see e.g. [14]. Explaining both the $(g-2)_\mu$ discrepancy and the proton radius puzzle by new particles coupling mainly to muons seems an attractive perspective. It does however require a significant fine-tuning, especially for larger values of the conjectured new particle masses [8]. To test the electron-muon universality, it has been proposed by the MUon proton Scattering Experiment (MUSE) [15] to make a simultaneous measurement of both μp and ep elastic scattering, extracting the proton charge form factor from both measurements. Besides the plans to measure μp elastic scattering, several new experiments are underway to extend ep scattering to lower momentum transfer values, down to 10^{-4} GeV^2 , and to cross-check its systematics [16, 17]. All of these tests require absolute cross section measurements, with a required precision on each absolute cross section at the level of 1 % or better.

In order to reach in μ scattering experiments the precision on R_E comparable with e^- scattering experiments, indicated in Eq. (2), we propose in this work a new experimental avenue through a relative cross section measurement of the photoproduction of e^-e^+ versus $\mu^-\mu^+$ pairs on a proton target. Besides being a well studied process [18], the photo-production of a lepton pair has the advantage that one produces e^-e^+ and $\mu^-\mu^+$ final states with the same beam, and thus the overall

normalization uncertainty drops out of their ratio. The analysis presented in this work shows that through a detection of the recoiling proton in the $\gamma p \rightarrow l^- l^+ p$ reaction, a measurement of the ratio of the e^-e^+ cross section below $\mu^-\mu^+$ threshold versus the $e^-e^+ + \mu^-\mu^+$ cross section sum above $\mu^-\mu^+$ threshold with an absolute precision of around 7×10^{-4} will allow to distinguish, at the 3σ level, between the proton R_E extractions from muonic and electronic observables. Furthermore, we show that the linear photon polarization asymmetry has a discriminatory power between the e^-e^+ and $\mu^-\mu^+$ channels in the $\mu^-\mu^+$ threshold region.

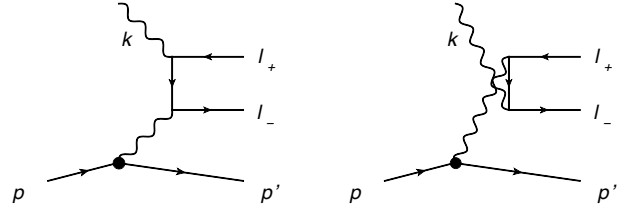


FIG. 1: Bethe-Heitler mechanism to the $\gamma p \rightarrow l^- l^+ p$ process, where the four-momenta of the external particles are: k for the photon, $p(p')$ for initial (final) protons, and l_-, l_+ for the lepton pair.

We will consider the lepton pair production on a proton target, $\gamma p \rightarrow l^- l^+ p$, in the limit of very small momentum transfer, defined as $t = (p - p')^2$, with four-momenta as indicated on Fig. 1. Furthermore, we will use in the following the Mandelstam invariant $s = (k + p)^2 = M^2 + 2ME_\gamma$, with M the proton mass and E_γ the photon *lab* energy, as well as the squared invariant mass of the lepton pair, defined as $M_{ll}^2 \equiv (l_- + l_+)^2$. In the limit of small $-t$, the Bethe-Heitler (BH) mechanism, shown in Fig. 1 totally dominates the cross section of the $\gamma p \rightarrow l^- l^+ p$ reaction. As the momentum transfer t is the argument appearing in the form factor (FF) in the BH process, a measurement of the cross section in this kinematic regime will allow to access the proton electric FF G_{Ep} at small spacelike momentum transfer, with a very small contribution of the proton magnetic FF G_{Mp} . As the differential cross section for the BH process is strongly peaked for leptons emitted in the incoming photon direction, and as we aim to maximize the BH contribution in this work in order to access G_{Ep} , we will study the $\gamma p \rightarrow l^- l^+ p$ process when (only) detecting the recoiling proton's momentum and angle, thus effectively integrating over the large lepton peak regions. The *lab* momentum of the proton is in one-to-one relation with

the momentum transfer t : $|\vec{p}'|^{lab} = 2M\sqrt{\tau(1+\tau)}$, with $\tau \equiv -t/(4M^2)$. Furthermore, for a fixed value of t , the recoiling proton lab angle Θ_p^{lab} is expressed in terms of invariants as :

$$\cos \Theta_p^{lab} = \frac{M_{ll}^2 + 2(s + M^2)\tau}{2(s - M^2)\sqrt{\tau(1+\tau)}}. \quad (3)$$

The differential cross section for the dominating BH process to the $\gamma p \rightarrow l^- l^+ p$ reaction has been studied in different contexts in the literature [18–20]. In this work, we will consider the cross section differential in the momentum transfer t and invariant mass of the lepton pair M_{ll}^2 , and integrated over the lepton angles, which corresponds with detecting the recoiling proton only. This cross section can be written as :

$$\begin{aligned} \frac{d\sigma^{BH}}{dt dM_{ll}^2} &= \frac{\alpha^3}{(s - M^2)^2} \cdot \frac{4\beta}{t^2(M_{ll}^2 - t)^4} \cdot \frac{1}{1 + \tau} \\ &\times \{C_E G_{Ep}^2 + C_M \tau G_{Mp}^2\}, \end{aligned} \quad (4)$$

with $\alpha \equiv e^2/4\pi \approx 1/137$, where $\beta \equiv \sqrt{1 - \frac{4m^2}{M_{ll}^2}}$ is the lepton velocity in the $l^- l^+$ *c.m.* frame, with m the lepton mass, and where the proton FFs G_{Ep} and G_{Mp} are functions of t . The weighting coefficients multiplying the FFs in Eq. (4) have the following general structure :

$$C_{E,M} = C_{E,M}^{(1)} + C_{E,M}^{(2)} \frac{1}{\beta} \ln \left(\frac{1 + \beta}{1 - \beta} \right), \quad (5)$$

where the second term expresses the large logarithmic enhancement in the limit of small lepton mass in the BH process. The coefficients $C_{E,M}^{(1)}$, and $C_{E,M}^{(2)}$ are found to be expressed through invariants as :

$$\begin{aligned} C_E^{(1)} &= t(s - M^2)(s - M^2 - M_{ll}^2 + t) [M_{ll}^4 + 6M_{ll}^2 t + t^2 + 4m^2 M_{ll}^2] \\ &+ (M_{ll}^2 - t)^2 [t^2 M_{ll}^2 + M^2(M_{ll}^2 + t)^2 + 4m^2 M^2 M_{ll}^2], \end{aligned} \quad (6)$$

$$\begin{aligned} C_E^{(2)} &= -t(s - M^2)(s - M^2 - M_{ll}^2 + t) [M_{ll}^4 + t^2 + 4m^2(M_{ll}^2 + 2t - 2m^2)] \\ &+ (M_{ll}^2 - t)^2 [-M^2(M_{ll}^4 + t^2) + 2m^2(-t^2 - 2M^2 M_{ll}^2 + 4m^2 M^2)], \end{aligned} \quad (7)$$

$$C_M^{(1)} = C_E^{(1)} - 2M^2(1 + \tau)(M_{ll}^2 - t)^2 [M_{ll}^4 + t^2 + 4m^2 M_{ll}^2], \quad (8)$$

$$C_M^{(2)} = C_E^{(2)} + 2M^2(1 + \tau)(M_{ll}^2 - t)^2 [M_{ll}^4 + t^2 + 4m^2(M_{ll}^2 - t - 2m^2)]. \quad (9)$$

In Fig. 2, we show the differential cross section $d\sigma/dt dM_{ll}^2$ for $\gamma p \rightarrow (l^- l^+) p$ which is accessed by measuring the recoiling proton's momentum and angle, for $E_\gamma = 0.5$ GeV and for three values of $-t$: $-t = 0.01$ GeV² (corresponding with recoil proton momentum $|\vec{p}'|^{lab} = 100$ MeV/c), $-t = 0.02$ GeV² ($|\vec{p}'|^{lab} = 142$ MeV/c), and $-t = 0.03$ GeV² ($|\vec{p}'|^{lab} = 174$ MeV/c). As t is the argument entering the proton FFs, the values shown are chosen to cover the lower range of the high-precision elastic ep scattering experiments [4, 5], as well as the values for which the future MUSE elastic μp scattering experiment [15] plans to take data. Furthermore, the differential cross section $d\sigma/dt dM_{ll}^2$ is shown as function of the squared lepton invariant mass M_{ll}^2 , which is dialed through the recoiling proton lab angle, as shown on the lower panel of Fig. 2. We show the cross section in a range of M_{ll}^2 , which is kinematically separated from background channels, well above the Compton and π^0 production processes on a proton, corresponding with sharp peaks at $M_{ll}^2 = 0$ and at $M_{ll}^2 = 0.018$ GeV² respectively, and below the threshold for $\pi\pi$ production, which starts at $M_{ll}^2 = 0.078$ GeV². One notices from Fig. 2 that around $M_{ll}^2 = 0.06$ GeV², the $\mu^- \mu^+$ cross section is around a factor of 10 smaller than the $e^- e^+$ cross section, and increases with increasing M_{ll}^2 . By measuring the cross section through detecting the recoiling proton momentum and angle in the M_{ll}^2 window above the π^0 peak

and below $\pi\pi$ threshold, and comparing cross sections at a fixed value of t above and below $\mu^- \mu^+$ thresholds, it opens the possibility for a high-precision extraction of the cross section ratio :

$$R_{\mu/e} \equiv \frac{d\sigma(\mu^- \mu^+ + e^- e^+)}{d\sigma(e^- e^+)}, \quad (10)$$

where $d\sigma$ stands for $d\sigma/dt dM_{ll}^2$. The potential advantage of such a ratio measurement is that absolute normalization uncertainties to first approximation drop out. Indeed, at a fixed value of t , the $e^- e^+$ cross section can be fixed by measuring the cross section below $\mu^- \mu^+$ threshold, and the corresponding normalization, due to G_{Ep} , can be used to determine the $e^- e^+$ cross section above $\mu^- \mu^+$ threshold. A subsequent measurement of the sum of $e^- e^+$ + $\mu^- \mu^+$ cross sections above $\mu^- \mu^+$ threshold, then allows to extract the ratio $R_{\mu/e}$, which is displayed in Fig. 3. One sees that in the kinematic range where only the $e^- e^+$ and $\mu^- \mu^+$ channels are contributing, this ratio varies between 1.10 to 1.14. We like to notice that corrections, notably radiative corrections, to first order also drop out of this ratio, measured at the same value of the recoiling proton momentum and angle. An accurate measurement of this ratio can therefore be envisaged, opening a new perspective to perform a test of lepton universality. We have demonstrated this sensitivity in Fig. 3, by

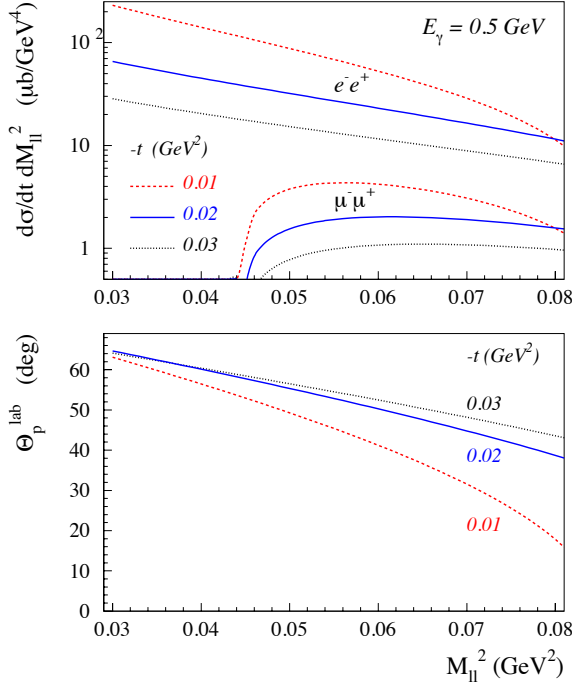


FIG. 2: Upper panel: comparison of the (lepton pair) invariant mass dependence of the $\gamma p \rightarrow e^-e^+p$ process (upper three curves) vs the $\gamma p \rightarrow \mu^-\mu^+p$ process (lower three curves) at $E_\gamma = 0.5 \text{ GeV}$, and for three values of the momentum transfer t as indicated. The lower panel shows the corresponding kinematic relation between the lepton pair invariant mass and the recoiling proton lab angle.

varying the electric FF value entering the e^-e^+ production process, denoted by G_{Ep}^e , which we take from [4, 5], versus the electric FF value entering the $\mu^-\mu^+$ production process, denoted by G_{Ep}^μ . We compare the case of lepton universality $G_{Ep}^\mu = G_{Ep}^e$, with a lepton universality violation scenario in which $G_{Ep}^\mu/G_{Ep}^e = 1.01$. The latter value is motivated by the around 1% larger value of G_{Ep}^μ at $-t = 0.03 \text{ GeV}^2$, resulting from the proton radius as extracted from the muonic hydrogen Lamb shift of Eq. (1), in comparison with the proton radius as extracted from ep scattering or electronic hydrogen Lamb shift, given by Eq. (2). We like to note that more recent studies, using a more conservative treatment of systematic errors in elastic ep scattering, extract a radius in agreement with Eq. (2), but with twice larger error [21].

We see from Fig. 3 that these two scenarios for the proton radius lead to a difference in the cross section ratio $R_{\mu/e}$ of Eq. (10) of around 2×10^{-3} . A measurement of this ratio $R_{\mu/e}$ with a precision of around 7×10^{-4} would therefore provide a test of such difference at the 3σ level. In Fig. 3, we show the 1, 3, and 5σ error bands corresponding with a σ of 7×10^{-4} . Although the Compton and π^0 photoproduction backgrounds have been eliminated by our choice of the kinematical range in $M_{||}^2$, we also estimated the background contamination due to the $\gamma p \rightarrow (\pi^0\gamma)p$ process [22] in the relevant range $M_{||}^2 \approx 0.07 \text{ GeV}^2$. We found that in order for

the contamination on $R_{\mu/e}$ to be bounded by $\Delta R_{\mu/e} \leq 10^{-3}$, the required precision on the $\gamma p \rightarrow \pi^0\gamma p$ cross section should be in the 10 - 30 % range, which has already been achieved by present experiments. Furthermore, the remaining concern is the physical background due to the indistinguishable time-like Compton scattering process, which results in the same final state. We estimate this timelike Compton process at the relatively low energy and momenta considered here by its Born contribution, corresponding to a nucleon intermediate state [23]. The inclusion of the timelike Compton Born contribution is also shown in Fig. 3. We found the effect due to the Born contribution in most kinematics around a factor of 5 smaller than the effect due to the 1 % variation in the value of G_{Ep}^μ , shown in Fig. 3. Although the timelike Compton process requires further study, we like to notice that it is not totally unknown, and a moderate accuracy in the 30 % range would be sufficient for its contamination to be bounded by $\Delta R_{\mu/e} \leq 10^{-3}$. We are therefore confident that the ratio of Eq. (10) is a promising observable for a lepton universality test at the 1 % level between G_{Ep}^μ and G_{Ep}^e . Besides providing such a test, a measurement of the absolute $\gamma p \rightarrow (e^-e^+)p$ cross section below $\mu^-\mu^+$ threshold will provide a useful cross-check on the extraction of G_{Ep} from elastic ep scattering. As the systematics in both reactions are quite different, an independent measurement of G_{Ep} may yield a valuable further clue to shed light on the “proton radius puzzle”.

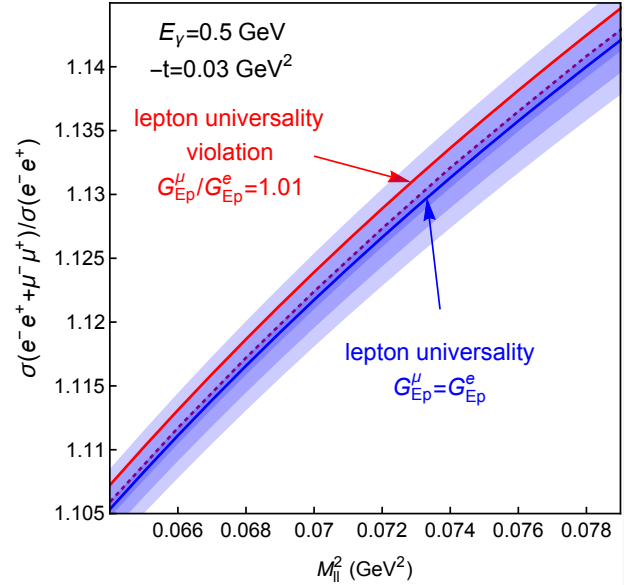


FIG. 3: Ratio $R_{\mu/e}$ of the $\gamma p \rightarrow (e^-e^+ + \mu^-\mu^+)p$ vs $\gamma p \rightarrow (e^-e^+)p$ differential cross sections, according to Eq. (10). The lower (blue) curve corresponds with the lepton universality result, for which $G_{Ep}^\mu = G_{Ep}^e$. The upper (red) curve assumes a violation of lepton universality, for which $G_{Ep}^\mu/G_{Ep}^e = 1.01$. The blue bands denote 1, 3, 5 σ bands corresponding with a σ of 7×10^{-4} around the blue curve. The difference between the blue curve and the dotted curve is an estimate of the physical background due to the contribution of the timelike Compton process.

Besides the unpolarized cross section, we may also consider the sensitivity of polarization observables to distinguish between the e^-e^+ and $\mu^-\mu^+$ production processes. We will

study here the case of the linear photon asymmetry defined as:

$$A_{lU} = \frac{d\sigma_{\parallel} - d\sigma_{\perp}}{d\sigma_{\parallel} + d\sigma_{\perp}}, \quad (11)$$

where $d\sigma_{\parallel}$ ($d\sigma_{\perp}$) stands for the differential cross section for a photon with linear polarization parallel (perpendicular) to the plane spanned by the photon and recoiling proton momenta. When measuring the recoiling proton only, the asymmetry above $\mu^{-}\mu^{+}$ threshold is given by the following weighted sum of the asymmetries of the $e^{-}e^{+}$ and $\mu^{-}\mu^{+}$ channels:

$$A_{lU}(e^{-}e^{+} + \mu^{-}\mu^{+}) = \frac{1}{R_{\mu/e}} \{A_{lU}(e^{-}e^{+}) + (R_{\mu/e} - 1) A_{lU}(\mu^{-}\mu^{+})\}. \quad (12)$$

We show in Fig. 4 the linear photon asymmetry in the kinematic range around $\mu^{-}\mu^{+}$ threshold. It is seen that the linear photon asymmetry is very small for the $e^{-}e^{+}$ channel. However, for the $\mu^{-}\mu^{+}$ channel the asymmetry reaches a value approaching -1 at $\mu^{-}\mu^{+}$ threshold and decreases in absolute value by going away from the threshold. Such behavior arises due to an exact cancellation, at $\mu^{-}\mu^{+}$ threshold, between the analytical and lepton mass logarithmic terms in the analogous expression as Eq. (5) for the contribution proportional to G_{Ep}^2 to σ_{\parallel} . In the whole M_{ll}^2 range of interest, the asymmetry for $\mu^{-}\mu^{+}$ production takes on large values as can be seen from Fig. 4. A direct measurement of the $\mu^{-}\mu^{+}$ asymmetry may therefore give a clear tool to separate the two channels. If the lepton pair is undetected, and only the recoiling proton is measured, the measured asymmetry above $\mu^{-}\mu^{+}$ threshold is diluted by the $\mu^{-}\mu^{+}/e^{-}e^{+}$ ratio, as given by Eq. (12), and reaches values around -5% as can be seen from Fig. 4.

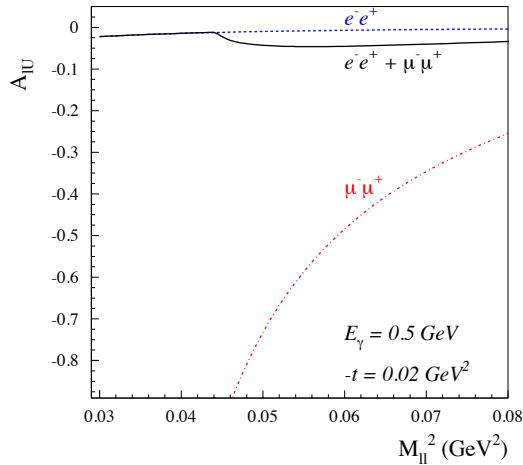


FIG. 4: Linear photon asymmetry A_{lU} of the $\gamma p \rightarrow (l^{-}l^{+})p$ process. The dashed (blue) curve corresponds with $e^{-}e^{+}$ production; the dashed-dotted (red) curve corresponds with $\mu^{-}\mu^{+}$ production. The solid (black) curve is the asymmetry corresponding with the sum of the $e^{-}e^{+} + \mu^{-}\mu^{+}$ channels according to Eq. (12).

In conclusion, in view of the sizably different proton charge radius presently extracted from electronic and muonic observables, we proposed a new lepton universality test which meets

the required precision goal to distinguish between them. We demonstrated that such a test is possible by comparing the photoproduction of a lepton pair on a proton, through detection of the recoiling proton. We showed that the measurement of the ratio of photoproduction cross sections of $e^{-}e^{+} + \mu^{-}\mu^{+}$ vs $e^{-}e^{+}$ pairs with an absolute precision of around 7×10^{-4} will allow to distinguish, at the 3σ level between the different proton R_E extractions from muonic and electronic observables. Such an experiment can be performed at existing electron facilities such as the Mainz Mikrottron (MAMI) and Jefferson Lab, thus adding a further piece of evidence towards an understanding of the “proton radius puzzle”.

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